



AnoMed

S-GBDT: Frugal Differentially Private Gradient Boosting Decision Trees

Moritz Kirschte[†], **Thorsten Peinemann[†]**, Joshua Stock^{*},
Carlos Cotrini[◇], Esfandiar Mohammadi[†].

The first two authors contributed equally to this work.

† Universität zu Lübeck * Universität Hamburg ◇ ETH Zürich



UNIVERSITÄT ZU LÜBECK
INSTITUT FOR IT SECURITY
PRIVACY & SECURITY GROUP



Finanziert von der
Europäischen Union
NextGenerationEU

GEFÖRDERT VOM

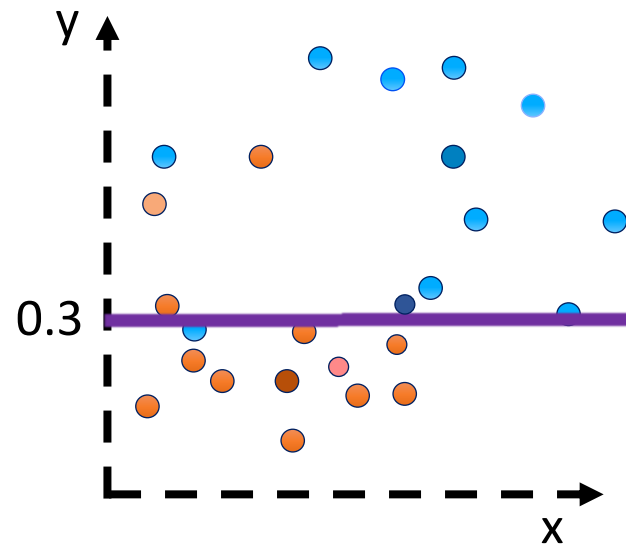


Bundesministerium
für Bildung
und Forschung

Regression using decision trees

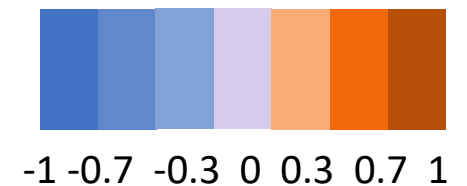
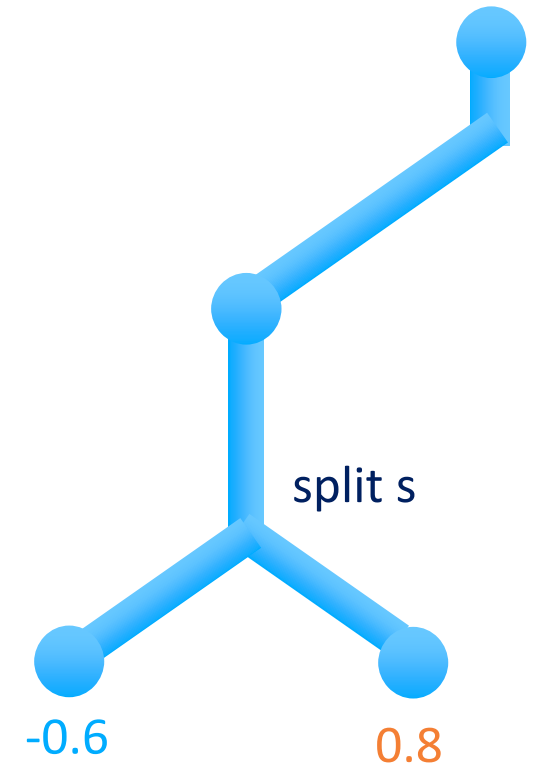
- Task: Regress data points (x, y : features)

- Step 1: Split



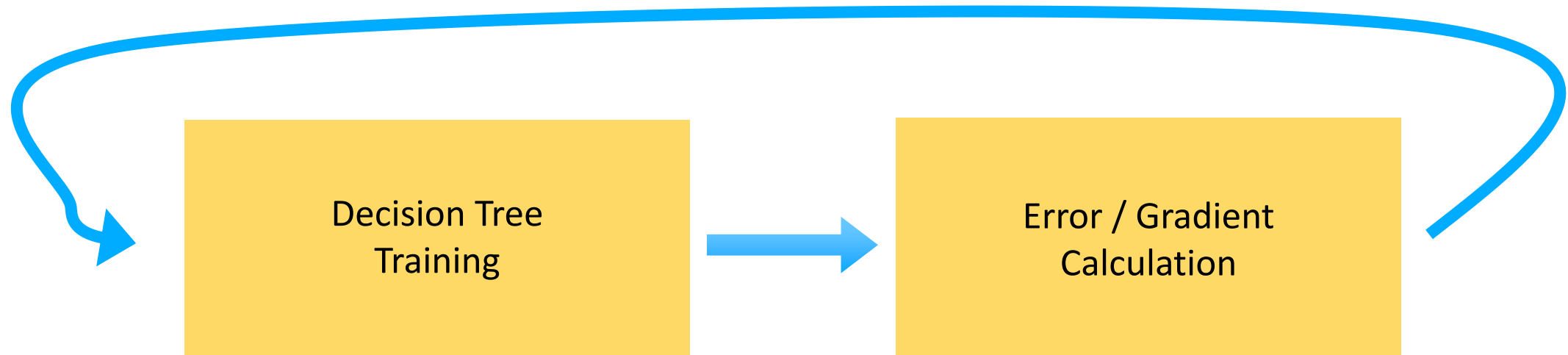
- Step 2: Predict

- If feature $y \geq 0.3$, then **blueish**, i.e. -0.6
- If feature $y < 0.3$, then **orangeish**, i.e. 0.8



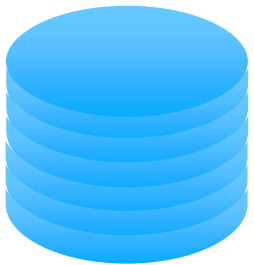
Gradient Boosting Decision Trees Ensemble

Training data:
Inputs (x,y) ; regression values / gradients

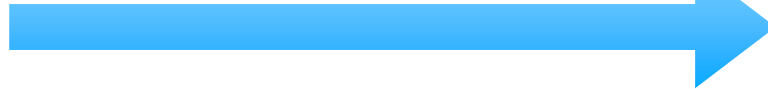


Tree-based ML applications can pose a privacy risk ^[1]

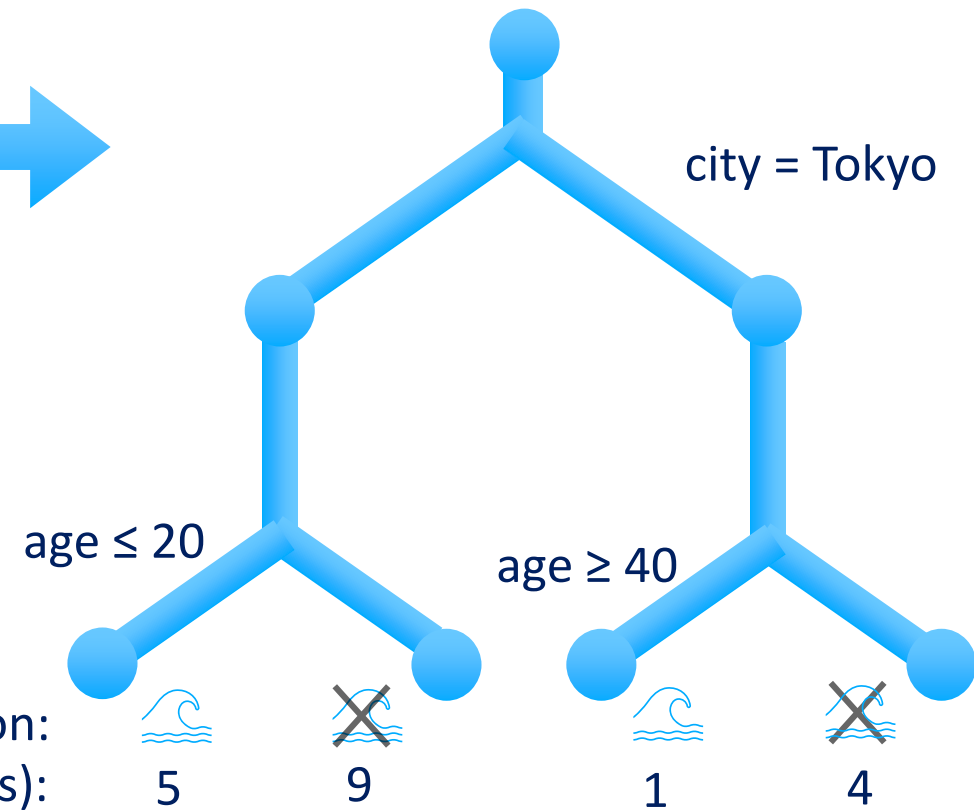
Training data
e. g. sensitive medical
survey data



Tree-based
Machine Learning (ML)

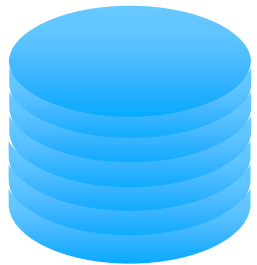


Public Name	Public City	Public Age	Sensitive Seasick?
Manu	Tokyo	42	
Nyasha	Hamburg	21	
...

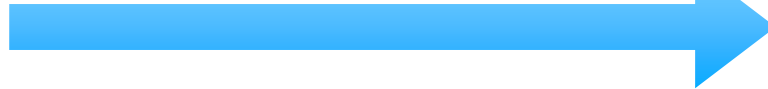


Tree-based ML applications can pose a privacy risk ^[1]

Training data
e. g. sensitive medical
survey data



Tree-based
Machine Learning (ML)

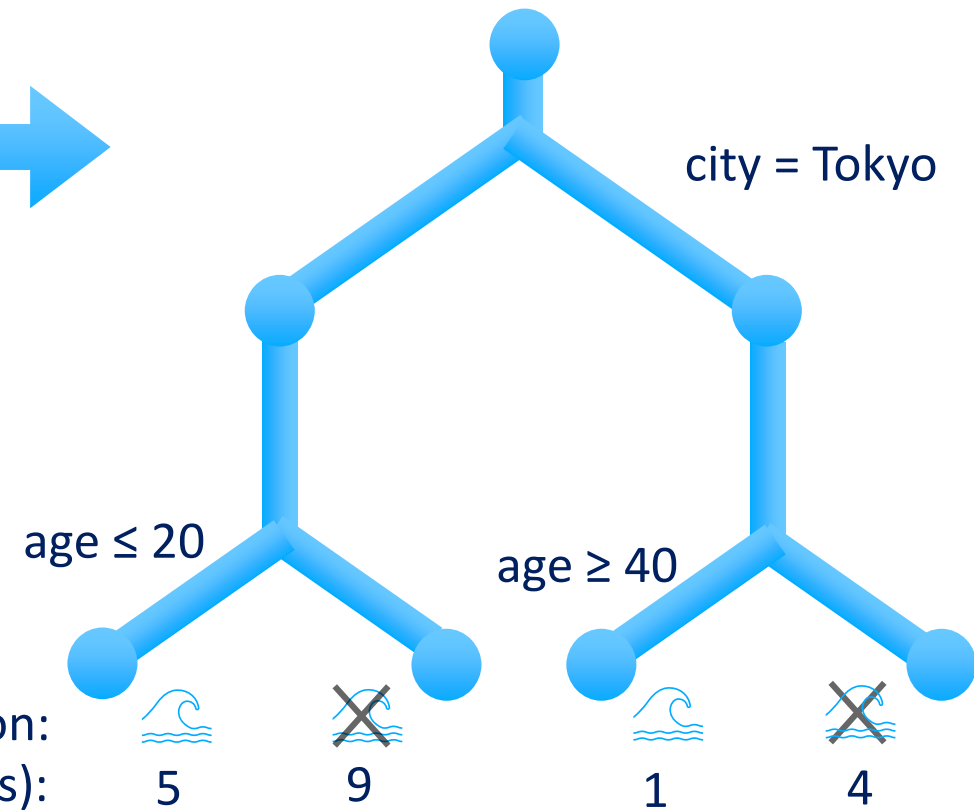


Privacy Attack
on tree

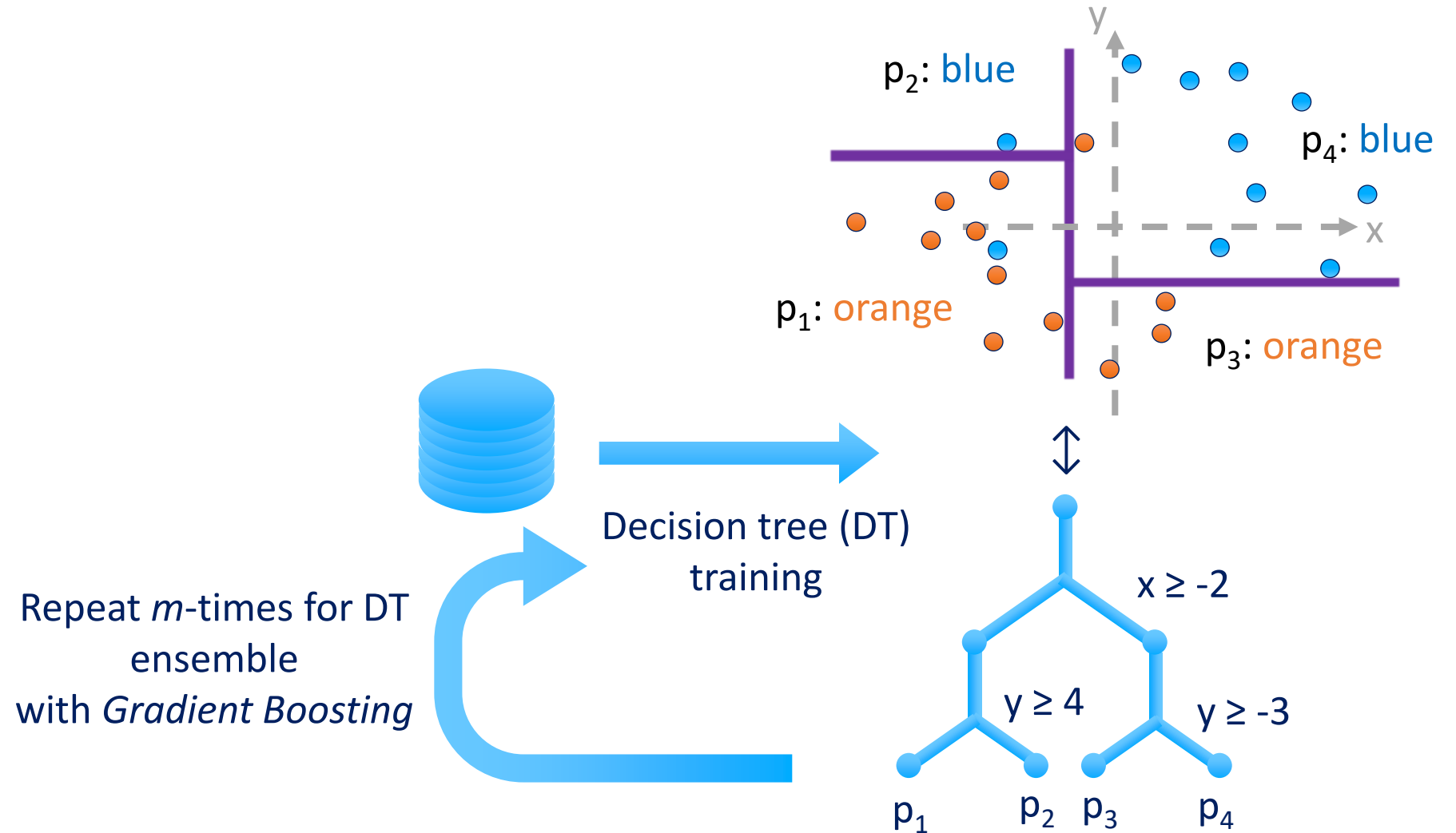


Attacker learns:
Manu must be
seasick.

Public Name	Public City	Public Age	Sensitive Seasick?
Manu	Tokyo	42	
Nyasha	Hamburg	21	
...



Prior work [2] counters tree-based privacy attacks as follows and sacrifices utility

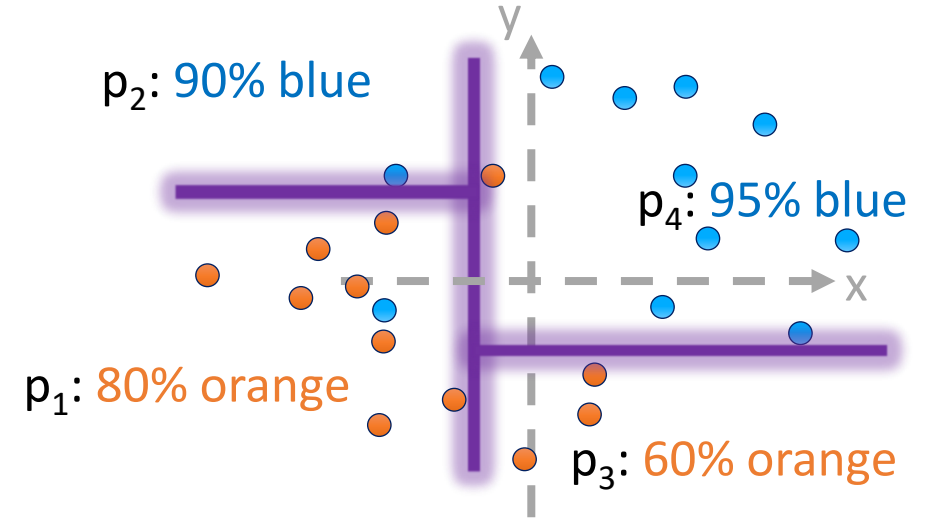


Prior work [2] counters tree-based privacy attacks as follows and sacrifices utility

1. Noise optimal split



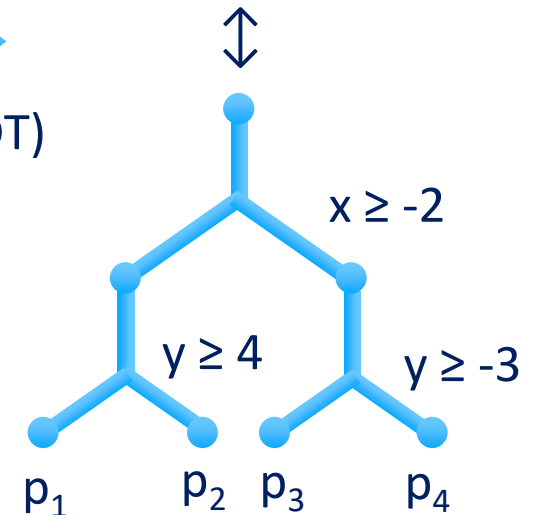
2. Noise leaf predictions



Repeat m -times for DT ensemble with *Gradient Boosting*

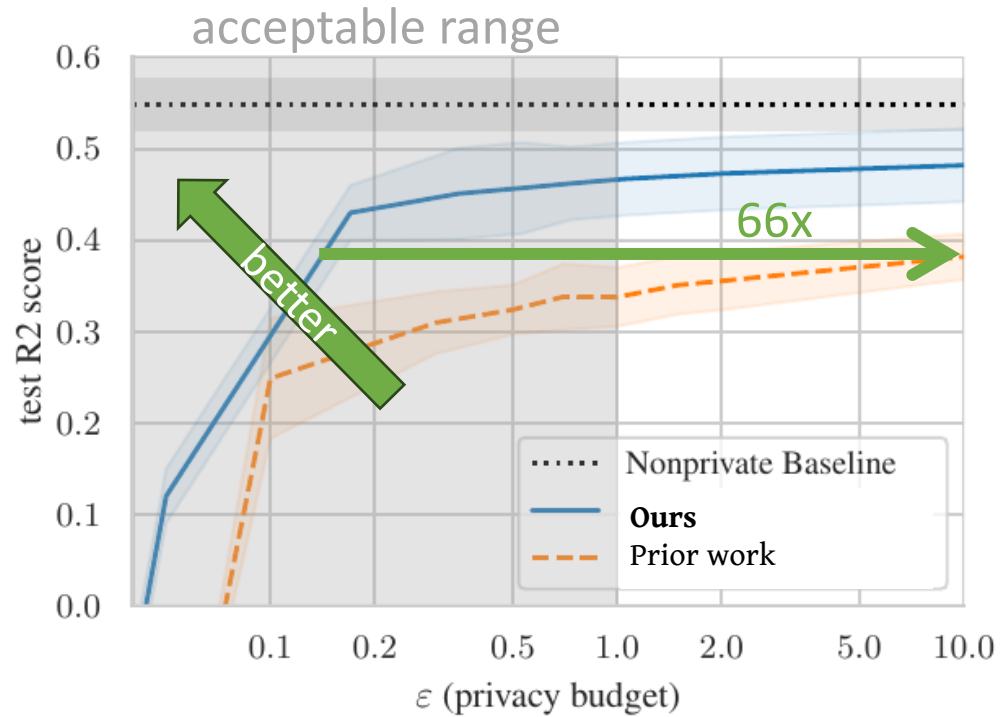


Decision tree (DT) training

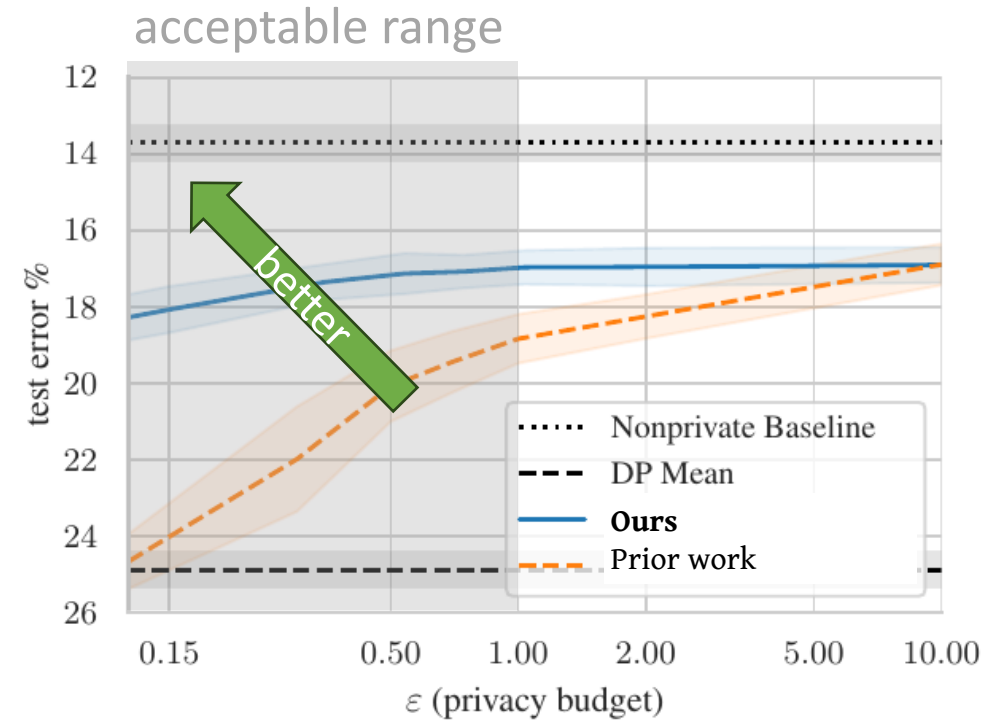


 : 2 classes

Experimental Results



(a) Abalone (Regression, < 4k data points)



(b) Adult (Binary classification, 48k data points)

(ϵ, δ) -Differential Privacy

- State of the art notion for provable privacy protection
- Deflects many privacy attacks
- Requires that the impact of single data points is limited and deniable
- We consider unbounded DP (add/remove relationship)

$$\Pr[M(D) = o] \leq e^\epsilon \Pr[M(D \cup \{x\}) = o] + \delta$$

randomized algorithm

worst-case dataset and challenge element

$(\alpha, \rho(\alpha))$ -Rényi Differential Privacy

- Rényi divergence of order α for any two probability distributions P, Q

$$D_\alpha(P||Q) = \frac{1}{\alpha - 1} \log \int_{-\infty}^{\infty} \frac{P(x)^\alpha}{Q(x)^{\alpha-1}} dx$$

density of P at x
density of Q at x

- $(\alpha, \rho(\alpha))$ -Rényi DP

$$D_\alpha(M(D)||M(D \cup \{x\})) \leq \rho(\alpha)$$

randomized algorithm
worst-case dataset and challenge element

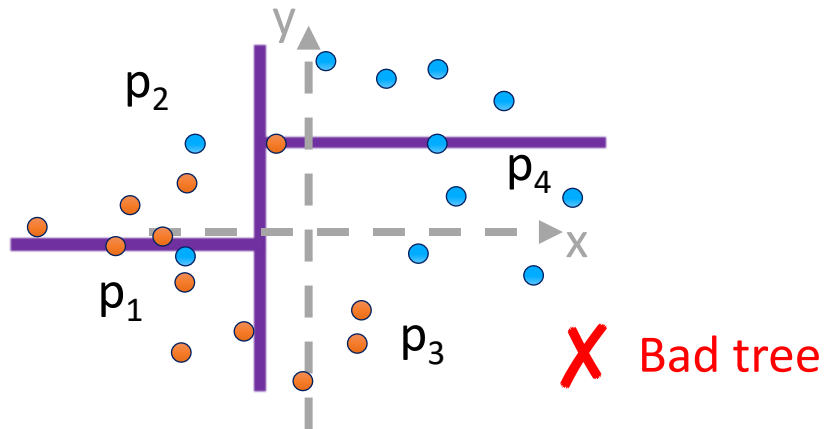
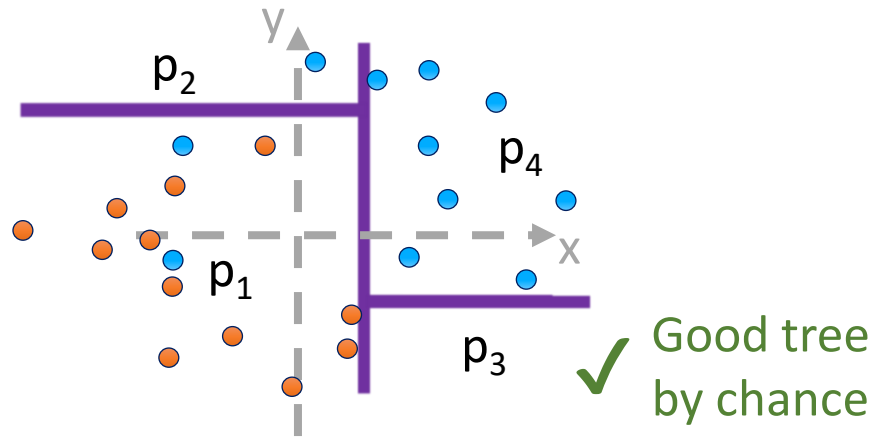
Our Improvements

Finally usable DP-GBDT!

Our Improvements

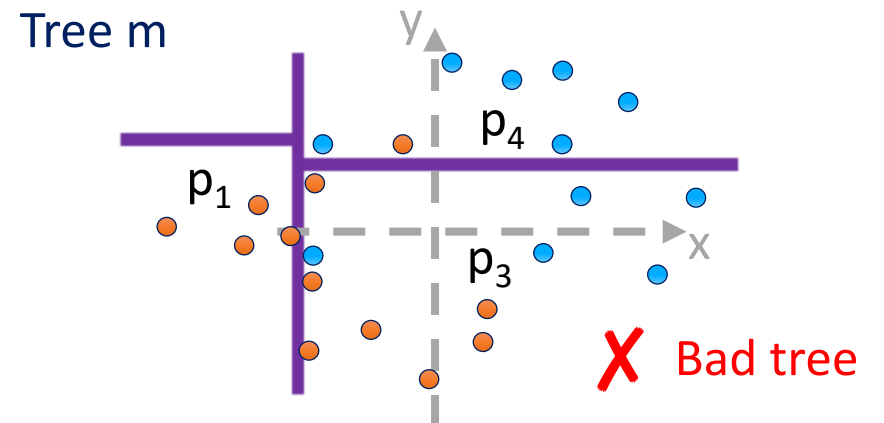
- **Rényi DP bound for Gaussian mechanism with non-spherical noise**
- **Dynamic leaf noise scaling**
- **Learning on streams of non-i.i.d. data via an individual Rényi filter**
- Incorporate random decision trees
- Subsampled tree learning with individual Rényi filter tailored to S-GBDT
- Extension for distributed learning
- Fixing secondary privacy leakages

Random splits enjoy little utility loss while being privacy protective



...

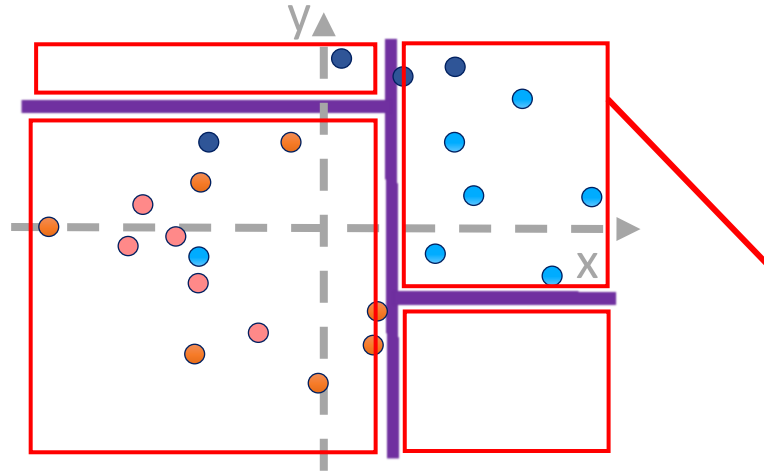
Tree 1 saves the ensemble.



In general: It suffices^[3] that only few trees of the ensemble are useful.

Differentially private leaf computation

 : data points, shade of color indicates gradient



Leaf value:

$$\frac{\sum_{i=1}^{|\text{leaf}|} \text{gradient}_i}{\max(\lambda, |\text{leaf}|)}$$

Prior work DP leaf value:

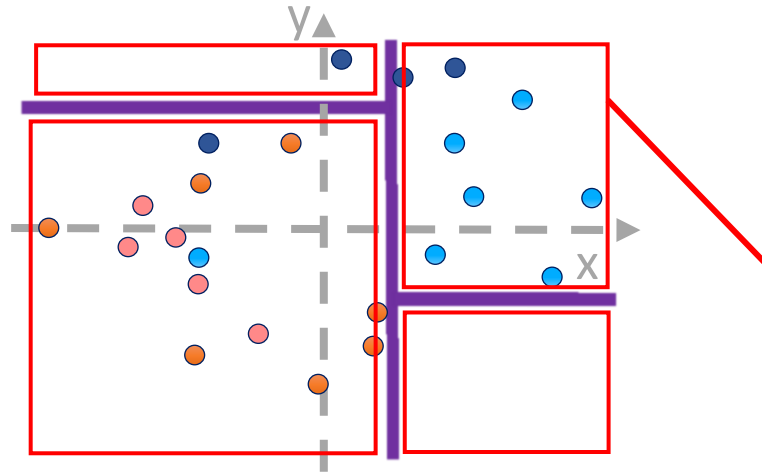
$$\frac{\sum_{i=1}^{|\text{leaf}|} \text{Clip}(\text{gradient}_i, g^*)}{\max(\lambda, |\text{leaf}|)} + \mathcal{N}\left(0, O\left(\frac{g^*}{\epsilon}\right)\right)$$

gradient sum

leaf support

Differentially private leaf computation

 : data points, shade of color indicates gradient



Leaf value:

$$\frac{\sum_{i=1}^{|\text{leaf}|} \text{gradient}_i}{\max(\lambda, |\text{leaf}|)}$$

Prior work DP leaf value:

$$\frac{\sum_{i=1}^{|\text{leaf}|} \text{Clip}(\text{gradient}_i, g^*)}{\max(\lambda, |\text{leaf}|)} + \mathcal{N}\left(0, O\left(\frac{g^*}{\epsilon}\right)\right)$$

sensitivity = g^*

sensitivity = 1

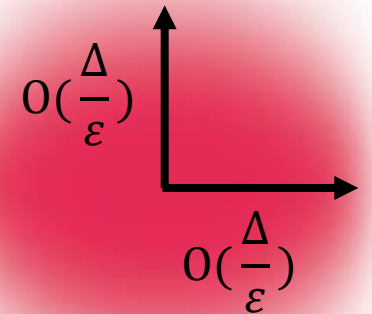
- ❖ Noise does not scale with number of data points n
- ❖ Privacy budget can not be shifted between gradient sum and leaf support

Dynamic leaf noise scaling

- Release both gradient sum and leaf support instead of averaged sum

DP Leaf value:
$$\frac{\sum_{i=1}^{|\text{leaf}|} \text{Clip}(\text{gradient}_i, g^*) + \mathcal{N}\left(0, O\left(\frac{\Delta}{\varepsilon}\right)\right)}{\max(\lambda, |\text{leaf}| + \mathcal{N}\left(0, O\left(\frac{\Delta}{\varepsilon}\right)\right))}$$

$$\Delta = \sqrt{1 + (g^*)^2}$$



Dynamic leaf noise scaling

Gradient sum divided by
(noisy) leaf support

Gradient sum noise divided by
(noisy) leaf support

DP Leaf value: $\frac{\sum_{i=1}^{|\text{leaf}|} \text{Clip}(\text{gradient}_i, g^*)}{\widetilde{\text{leaf}}} + \frac{\mathcal{N}\left(0, O\left(\frac{\Delta}{\varepsilon}\right)\right)}{\widetilde{\text{leaf}}}$

$$\Delta = \sqrt{1 + (g^*)^2} \quad \widetilde{\text{leaf}} = \max(\lambda, |\text{leaf}| + \mathcal{N}\left(0, O\left(\frac{\Delta}{\varepsilon}\right)\right))$$

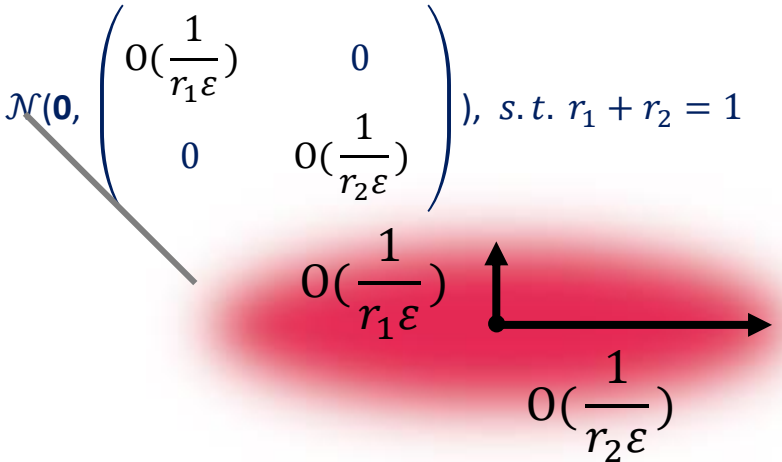
- ✓ Noise scales in $O\left(\frac{1}{n}\right)$ for number of data points n
- ❖ Privacy budget can not be shifted between gradient sum and leaf support

Non-spherical noise

Clipped leaf value:
$$\frac{\sum_{i=1}^{|\text{leaf}|} \text{Clip}(\text{gradient}_i, g^*)}{\max(\lambda, |\text{leaf}|)}$$

sensitivity = g^*

sensitivity = 1



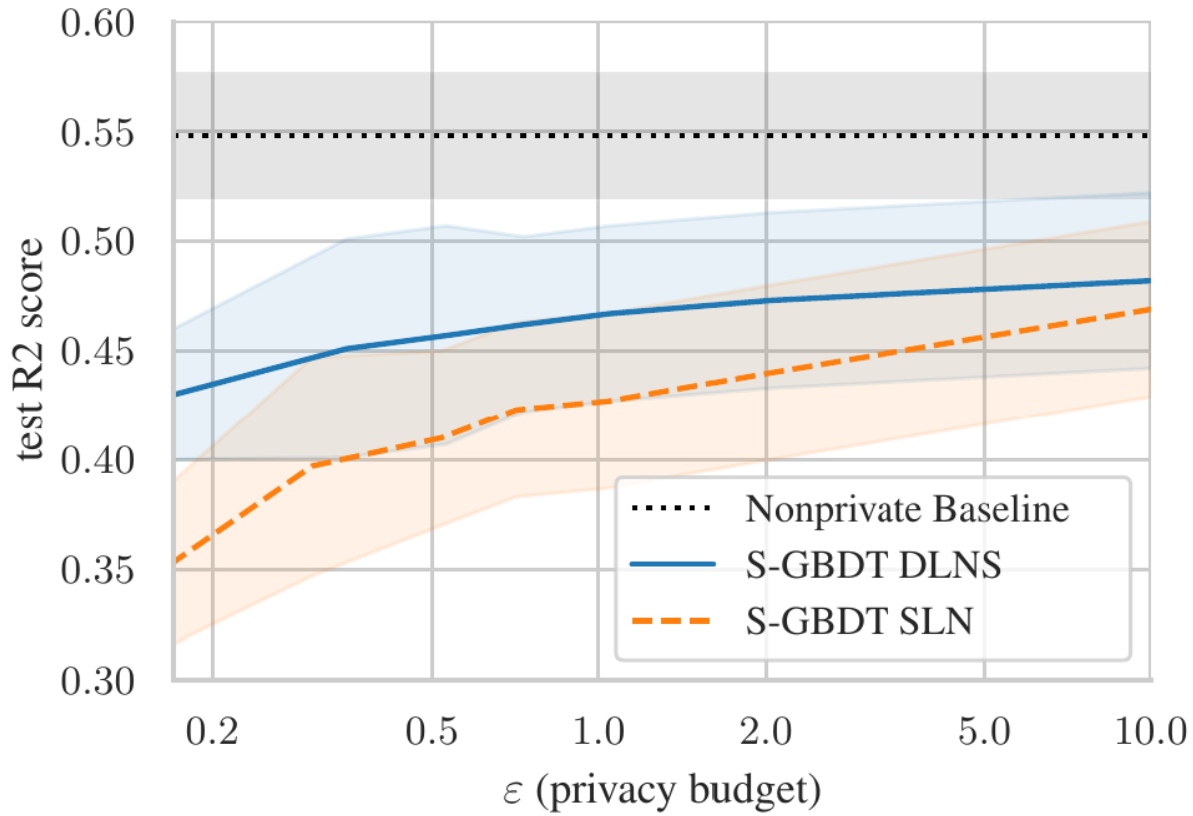
DP Leaf value:
$$\frac{\sum_{i=1}^{|\text{leaf}|} \text{Clip}(\text{gradient}_i, g^*) + \mathcal{N}\left(0, O\left(\frac{1}{r_1\epsilon}\right)\right)}{\max\left(\lambda, |\text{leaf}| + \mathcal{N}\left(0, O\left(\frac{1}{r_2\epsilon}\right)\right)\right)}$$

RDP bound:
$$\rho(\alpha) = \alpha \frac{r_1 + r_2 * (g^*)^2}{\sigma^2}$$

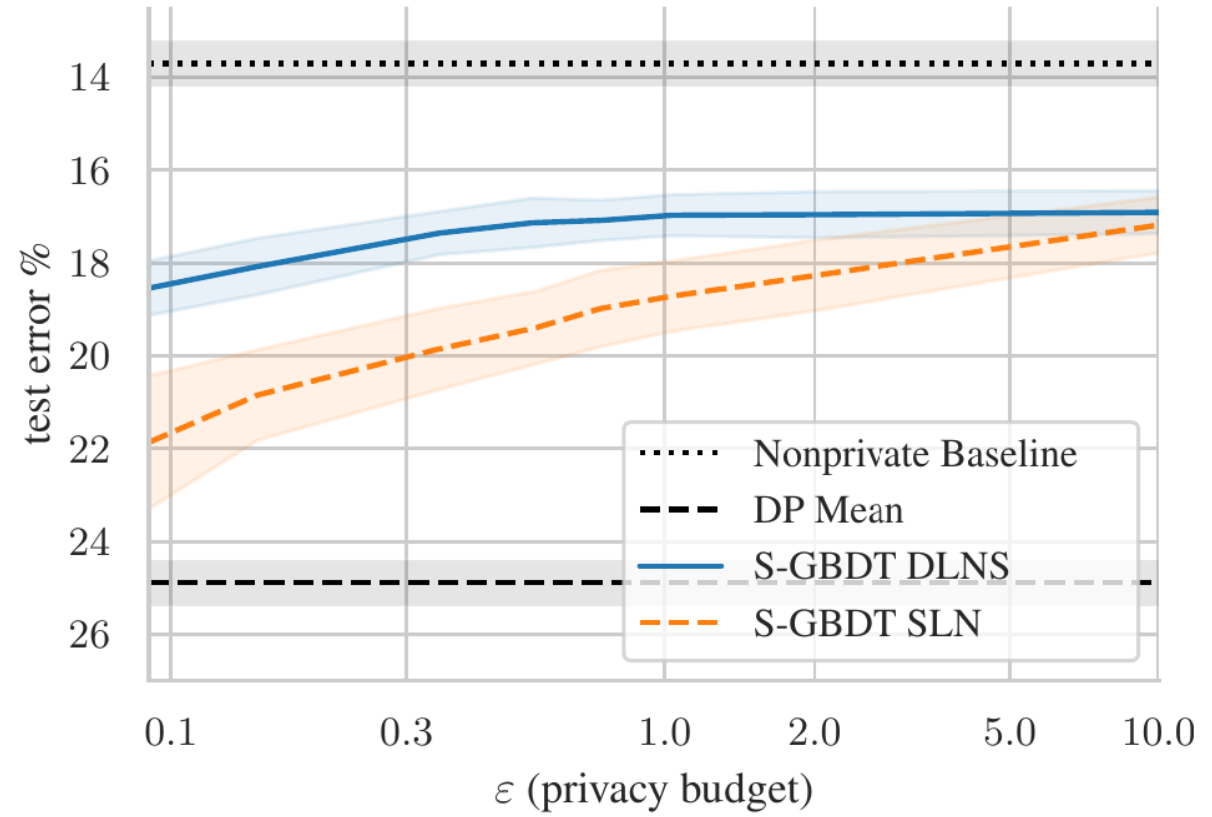
see paper for proof or ask me

✓ Privacy budget can be shifted between gradient sum and leaf support

Experimental Results



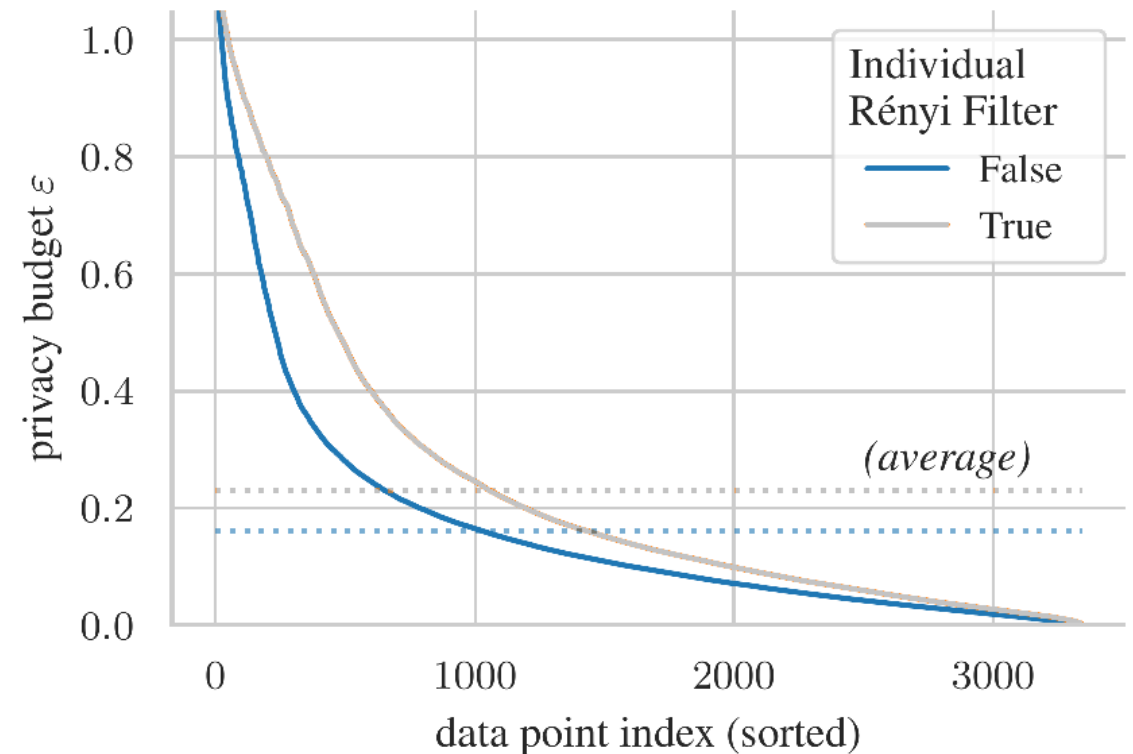
(a) **Abalone**



(b) **Adult**

Conventional Privacy Accounting

- Conventional approach for privacy accounting: same worst case analysis applied to all data points
- Can result in overly conservative estimation of privacy loss for many data points
- Data points in S-GBDT often do not fully utilize sensitivity (i.e. $g_i < g^*$)



Individual Rényi DP [4]

Individual $(\alpha, \rho(\alpha))$ -Rényi DP for data point \mathbf{x}_i : $D_\alpha(M(D) || M(D \cup \{\mathbf{x}_i\})) \leq \rho^{(i)}(\alpha)$

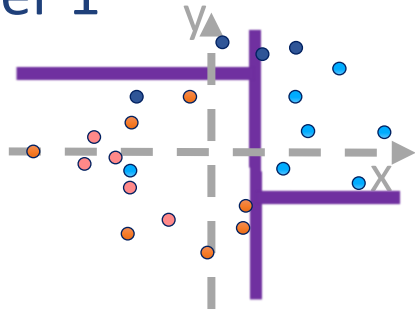
worst case dataset

Individual RDP bound for releasing leaf value: $\rho^{(i)}(\alpha) = \alpha \frac{r_1 + r_2 * (\mathbf{g}_i)^2}{\sigma^2}$

value has leakage when released,
but can be utilized by individual Rényi filter

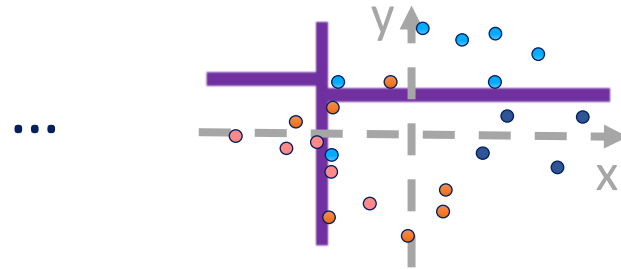
Tailoring individual Rényi filters ^[4] to S-GBDT

Classifier 1



Datapoint	Budget
$(x_1, y_1, \text{label}_1)$	0%
$(x_2, y_2, \text{label}_2)$	0%
...	...

Classifier m



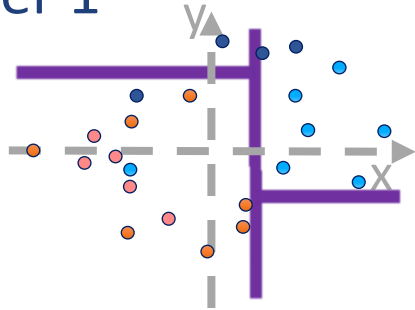
Datapoint	Budget
$(x_1, y_1, \text{label}_1)$	50%
$(x_2, y_2, \text{label}_2)$	90%
...	...

individual Rényi divergence budget $\rho^{(i)}(\alpha)$

Upper bound on RDP privacy loss for m trees: $m * \rho(\alpha)$

Tailoring individual Rényi filters ^[4] to S-GBDT

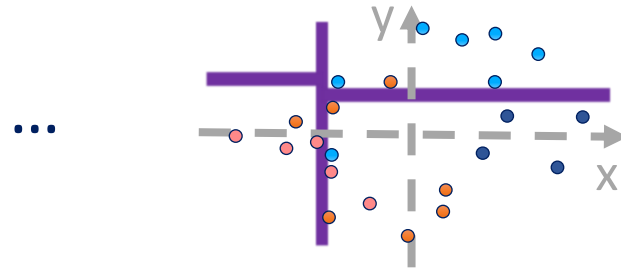
Classifier 1



Datapoint	Budget
$(x_1, y_1, \text{label}_1)$	0%
$(x_2, y_2, \text{label}_2)$	0%
...	...

individual Rényi divergence budget $\rho^{(i)}(\alpha)$

Classifier m

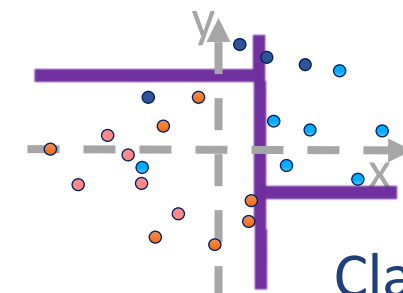


Datapoint	Budget
$(x_1, y_1, \text{label}_1)$	50%
$(x_2, y_2, \text{label}_2)$	90%
...	...

Upper bound on RDP privacy loss for m trees: $m * \rho(\alpha)$

Regular rounds

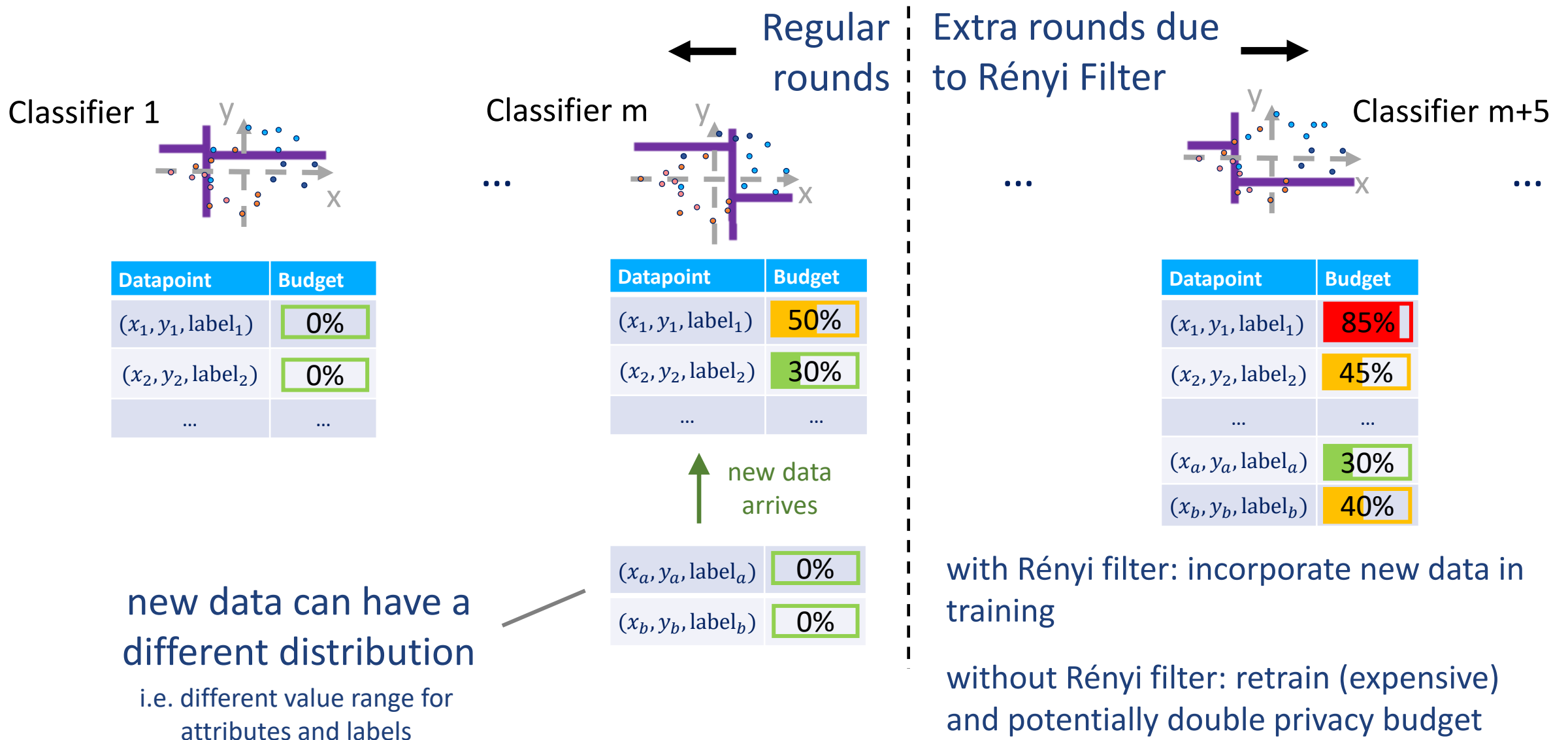
Extra rounds due to Rényi Filter



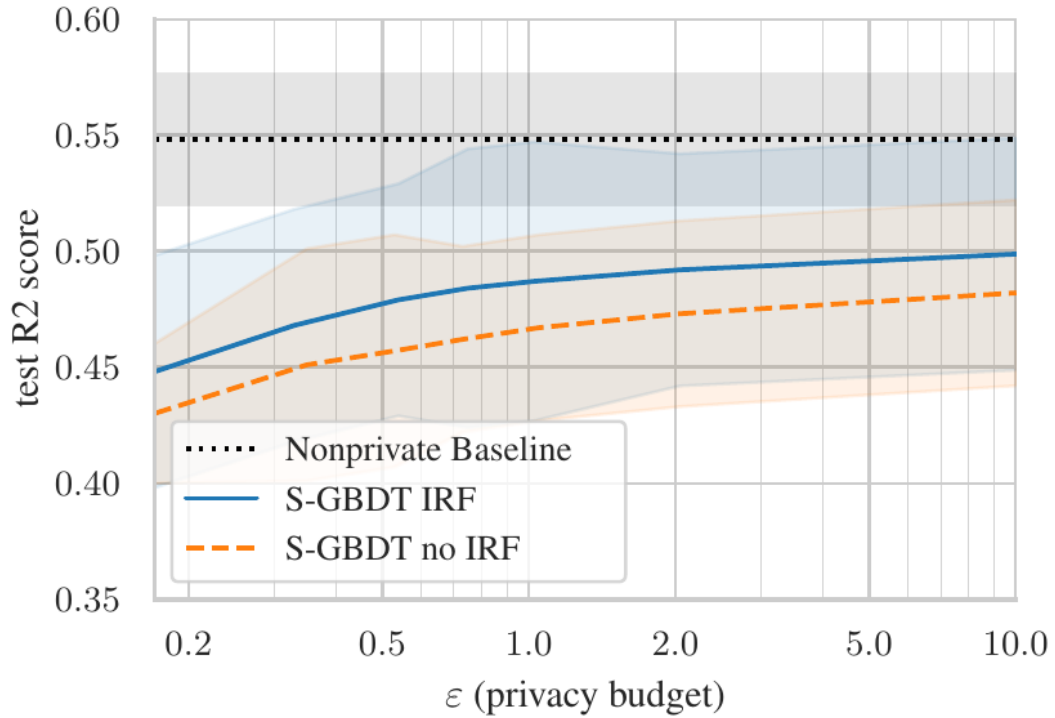
Datapoint	Budget
$(x_1, y_1, \text{label}_1)$	75%
$(x_2, y_2, \text{label}_2)$	100%
...	...

Excluded from training

Insight: Individual Rényi filters ^[4] are effective for streams of non-i.i.d. Data

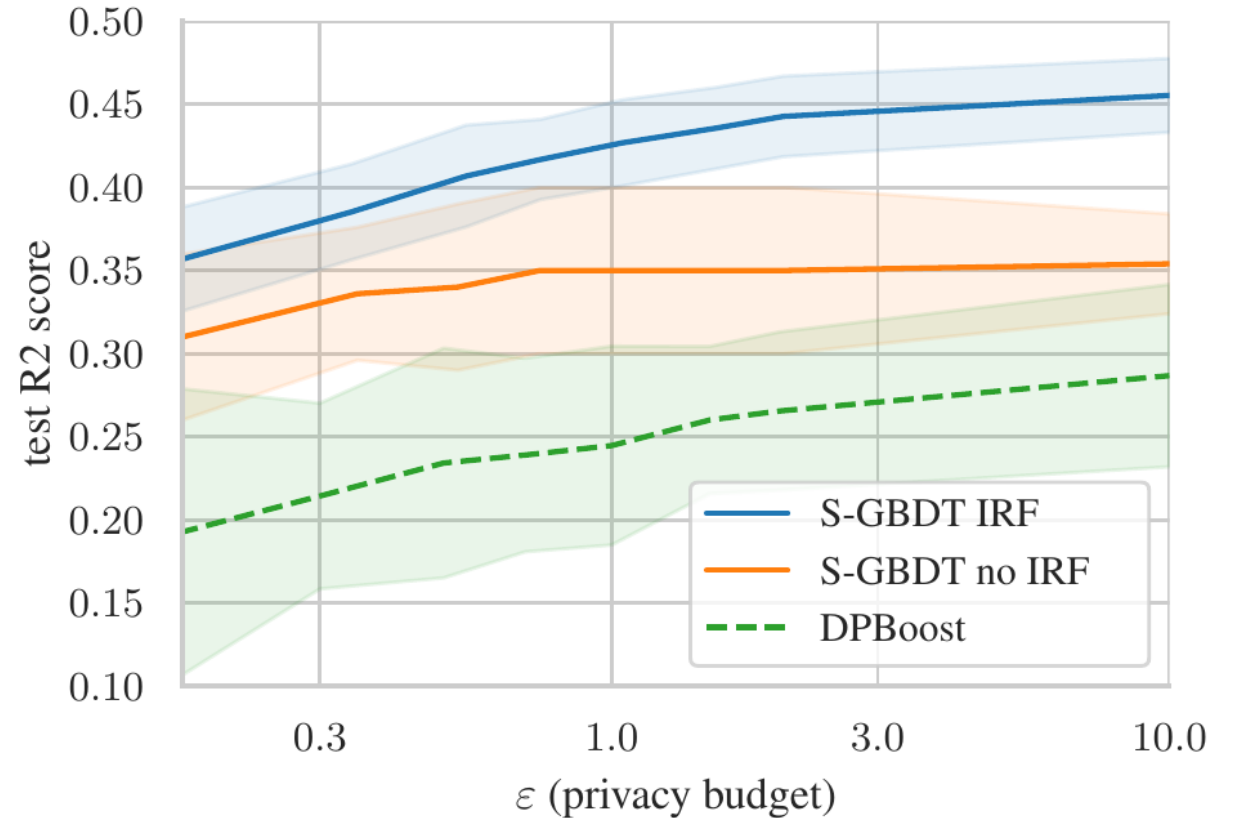


Experimental Results



(a) **Abalone**

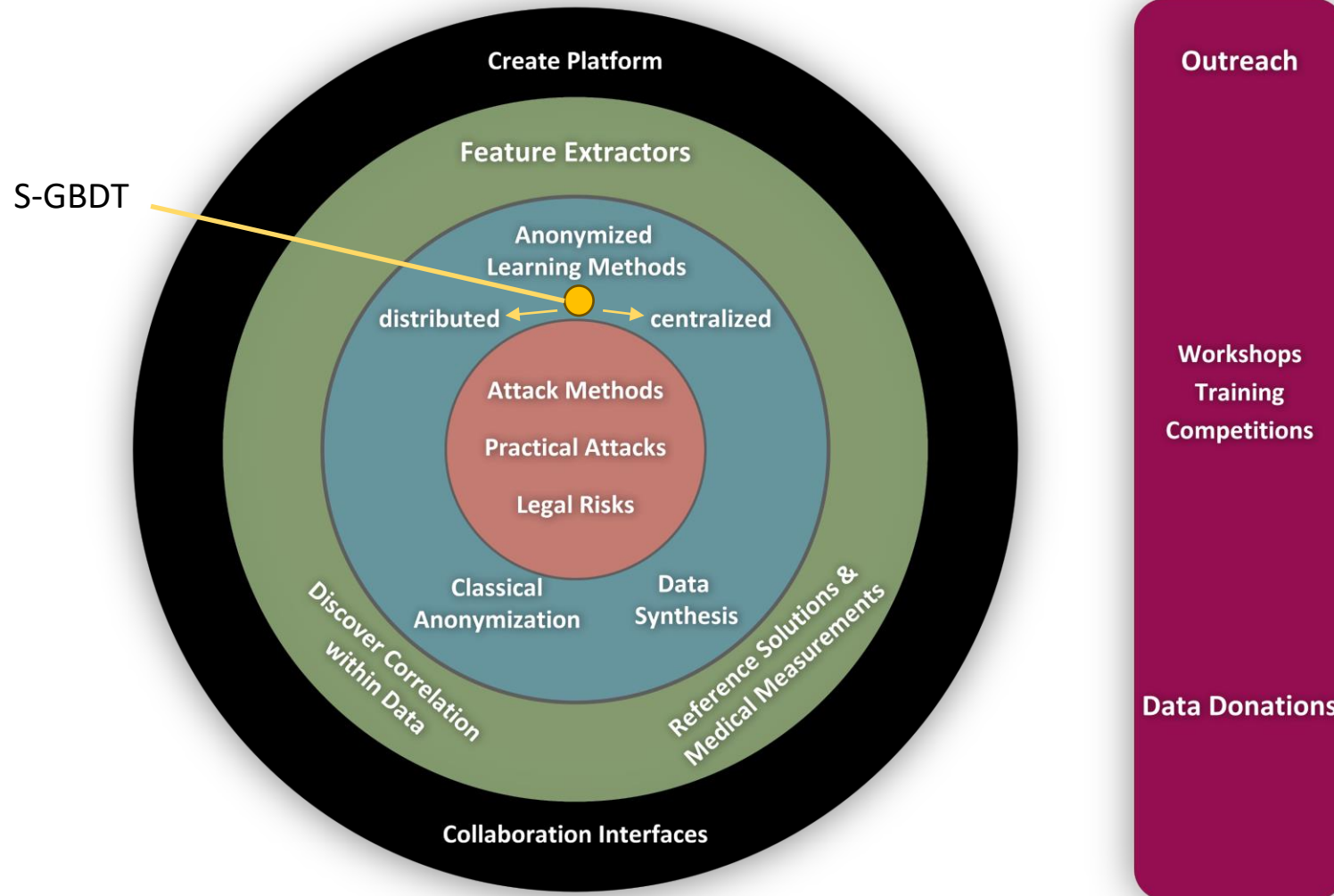
regular training



(a) **Abalone**

learning on streams

Contribution to AnoMed



Full version on arXiv

